

Maxwell's demons in multipartite quantum correlated systems

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(Dated: December 10, 2014)

We investigate the extraction of thermodynamic work by a Maxwell's demon in a multipartite quantum correlated system. We begin by adopting the standard model of a Maxwell's demon as a Turing machine, either in a classical or quantum setup depending on its ability of implementing classical or quantum conditional dynamics, respectively. Then, for an n -partite system (A_1, A_2, \dots, A_n) , we introduce a protocol of work extraction that bounds the advantage of the quantum demon over its classical counterpart through the amount of multipartite quantum correlation present in the system, as measured by a thermal version of the global quantum discord. This result is illustrated for an arbitrary n -partite pure state of qubits with Schmidt decomposition, where it is shown that the thermal global quantum discord exactly quantifies the quantum advantage. Moreover, we also consider the work extraction via mixed multipartite states, where examples of tight upper bounds can be obtained.

PACS numbers: 03.67.-a, 03.67.Mn, 03.65.Ud

I. INTRODUCTION

The concept of Maxwell's demon has introduced a deep relationship between information theory and thermodynamics [1]. As originally proposed, the demon can be thought as a microscopic "intelligent" being capable of extracting work from a thermodynamic system at apparent no energy cost. As a simple example, consider a gas (initially in an equilibrium thermal state) contained in a chamber divided into two parts by an insulated wall. By direct inspection (followed by a post-selection) of fast particles, the demon would then be able to create a temperature gradient between the two parts, which could be used, e.g., as an energy resource for a thermal machine. Naturally, in order to provide a continuous work extraction, a cyclic process must be required, which imposes the erasure of the demon's memory for a complete thermodynamic accounting. Indeed, the irreversibility of the erasure operation, which is the main content of the Landauer principle [2], is the ultimate reason responsible for the conciliation of the Maxwell's demon with the second law of thermodynamics.

In a modern perspective, we can take a Maxwell's demon as any device with the ability of information processing (as a computer modeled by a Turing Machine), where the extraction of work comes at the only cost of memory erasure at the end of the process. Remarkably, it has been shown by Zurek in Ref. [3] that a quantum demon, which has the ability of implementing a quantum conditional dynamics through global operations over the system, can be more efficient in extracting work of a quantum system than any classical demon, which acts

through local operations and classical communication to implement a classical dynamics. In a bipartite system-apparatus scenario, this difference can be quantified by the amount of quantum correlations between system and apparatus, as measured by a thermal version of the quantum discord (QD) [3, 4]. Indeed, QD has been identified as a general resource in quantum information protocols (see, e.g., Refs. [5–8]). In quantum computation, it has been conjectured as the origin of speed up in the deterministic quantum computation with one qubit (DQC1) mixed-state model [9]. Moreover, remarkable applications of QD have also been found in the characterization of quantum phase transitions [10] and in the description of quantum dynamics under decoherence [11]. In this context, an operational interpretation of the QD in terms of the efficiency of a Maxwell's demon establishes a solid framework to investigate its conceptual role in quantum thermodynamics as well as to inspire new QD-based quantum protocols.

In this work, we aim at investigating the efficiency of both classical and quantum Maxwell's demons in the multipartite scenario. In particular, we are interested in analyzing the relationship between the quantum advantage and the existence of multipartite quantum correlations. More specifically, provided n copies of an n -partite system (A_1, A_2, \dots, A_n) , we introduce a protocol of work extraction defined through a sequence of intermediate steps, where the Maxwell demon (either classical or quantum) uses a subsystem A_i of copy i ($i = 1, \dots, n$) as a measurement apparatus at a each step, with i a sequential label chosen at demon's will. The demon is also required to erase its memory at the end of the process, so that the thermodynamic accounting does not disregard the irreversible local cost of erasure. In this context, we will show that the advantage of the quantum demon over its classical counterpart in extracting work (from the n copies) is bounded through the amount of multipartite quantum correlation, as measured by a thermal version

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of the global quantum discord (GQD) [12]. Indeed, GQD has been introduced as a multipartite approach to quantify quantum correlations, which has been applied to the characterization of quantum phase transitions in many-body systems [12, 13]. Moreover, it can be witnessed with no extremization procedure [14] and it has been used as a tool to define a monogamy relationship for the standard QD [15]. Here, we provide a generalization of GQD to a thermodynamical scenario as well as an operational interpretation of this thermal version of GQD. We illustrate our procedure of work extraction for an arbitrary n -partite pure state of qubits with Schmidt decomposition, where it is shown that the thermal GQD exactly quantifies the quantum advantage of the Maxwell's demon. Moreover, we also provide examples of work extraction via mixed multipartite states. These examples allow for the discussion of the tightness of the bound in situations where saturation is not always achieved.

II. MAXWELL'S DEMON AND BIPARTITE QUANTUM CORRELATIONS

Work and information are equivalent operational concepts [16] (see also, e.g., Refs. [17–19] for more recent discussions). If a system \mathcal{S} described by a d -level pure state $|\psi\rangle$ is available as a resource, we can extract work from \mathcal{S} by letting it expand throughout Hilbert space while in contact with a thermal reservoir at temperature T . For such an isothermal process, one can draw work $W = kT \log d$ out of the heat bath, where k is the Boltzmann constant adapted to deal with the entropy in bits (so that $\log \equiv \log_2$). For a mixed state ρ , less work is possible to be extracted, since less knowledge (information) about the state of the system is available. In this situation, we should discount the necessary work to be performed over the system to drive it to a pure state. This yields

$$W = kT [\log d - S(\rho)], \quad (1)$$

where $S(\rho) = -\text{Tr} \rho \log \rho$ is the von Neumann entropy associated with the quantum state ρ . In a bipartite system-apparatus (\mathcal{SA}) scheme, we can then write the work W^Q extracted by a quantum demon as

$$W^Q = kT [\log d_{SA} - S(\rho_{SA})], \quad (2)$$

where d_{SA} is the dimension of \mathcal{SA} and $S(\rho_{SA})$ is its joint von Neumann entropy. A classical demon, on the other hand, first implements a local measurement $\{\Pi_A^k\}$ on the apparatus, using the measured state to extract work $\log(d_A) - H(\{p_a\})$ from \mathcal{A} , where $H(\{p_a\}) = -\sum_a p_a \log p_a$ is the Shannon entropy for the probability distribution $\{p_a\}$ associated with the local measurement $\{\Pi_A^k\}$. Then, an update of \mathcal{S} is performed based on the outcome read from the apparatus and work $\log(d_S) - S(\rho_S|\{\Pi_A^k\})$ is extracted from \mathcal{S} , where $S(\rho_S|\{\Pi_A^k\})$ is the conditional entropy accessible through

$\{\Pi_A^k\}$, which is given by the weighted average

$$S(\rho_S|\{\Pi_A^k\}) = \sum_i p_i S(\rho_i), \quad (3)$$

with $S(\rho_i)$ denoting the von Neumann entropy of the post-measurement state $\rho_i = (1/p_i)(I_S \otimes \Pi_A^i)\rho_{SA}(I_S \otimes \Pi_A^i)$ and $p_i = \text{Tr}[(I_S \otimes \Pi_A^i)\rho_{SA}(I_S \otimes \Pi_A^i)]$. In this work, for simplicity, we will typically restrict Π_A^k as rank-one orthogonal projective measurements rather than arbitrary positive operator-valued measures (POVMs). The total amount of work extracted is then given by

$$W^C = kT [\log d_{SA} - S_A(\rho_{SA})], \quad (4)$$

where $S_A(\rho_{SA})$ is the locally accessible joint entropy, which reads

$$S_A(\rho_{SA}) = H(\{p_a\}) + S(\rho_S|\{\Pi_A^k\}). \quad (5)$$

Remarkable, the minimum difference between W^Q and W^C , which is given by the best classical strategy, can be quantified by the quantum correlation between \mathcal{S} and \mathcal{A} , as measured by the thermal QD. As defined in Ref. [3], the thermal QD $\mathcal{D}_{th}(S|A)$ for a composite system \mathcal{SA} can be suitably expressed (with respect to \mathcal{A}) as the difference between the quantum mutual information

$$I(\rho_{SA}) = S(\rho_S) + S(\rho_A) - S(\rho_{SA}) \quad (6)$$

and the locally accessible mutual information

$$J_A(\rho_{SA}) = S(\rho_S) + S(\rho_A) - S_A(\rho_{SA}), \quad (7)$$

with the difference $I(\rho_{SA}) - J_A(\rho_{SA})$ minimized over all local measurements $\{\Pi_A^k\}$. This reads

$$\mathcal{D}_{th}(S|A) = \min_{\{\Pi_A^k\}} [S_A(\rho_{SA}) - S(\rho_{SA})]. \quad (8)$$

Then, by using Eqs. (2), (4) and (8), it has been shown in Ref. [3] that

$$\Delta W \equiv \min_i (W^Q - W^{C_i}) = kT \mathcal{D}_{th}(S|A), \quad (9)$$

where \min_i denotes the minimum over the difference $(W^Q - W^{C_i})$ for all the possible strategies $\{\Pi_A^k\}$ for work extraction adopted by a classical demon C_i . In terms of conditional entropies, the thermal QD can be also written as

$$\mathcal{D}_{th}(S|A) = \min_{\{\Pi_A^k\}} \{ [H(\{p_a\}) + S(\rho_S|\{\Pi_A^k\})] - [S(\rho_A) + S(\rho_S|\rho_A)] \}, \quad (10)$$

with

$$S(\rho_S|\rho_A) = S(\rho_{SA}) - S(\rho_A) \quad (11)$$

denoting the entropy of \mathcal{S} conditional on \mathcal{A} . Therefore, the thermal QD $\mathcal{D}_{th}(S|A)$ is distinct of the original QD $\mathcal{D}(S|A)$ proposed in Ref. [4], which is given by

$$\mathcal{D}(S|A) = \min_{\{\Pi_A^k\}} [S(\rho_S|\{\Pi_A^k\}) - S(\rho_S|\rho_A)]. \quad (12)$$

In particular, the thermal QD is also referred as the one-way work deficit [20]. Moreover, it follows that $\mathcal{D}_{th}(S|A) \geq \mathcal{D}(S|A)$ [21].

III. MAXWELL'S DEMON AND MULTIPARTITE QUANTUM CORRELATIONS

Let us now present a thermodynamic protocol able to provide an operational interpretation for multipartite quantum correlations as measured by the thermal GQD. In order to introduce the thermal GQD, let us first rewrite $\mathcal{D}_{th}(S|A)$ in terms of loss of total correlation after a non-selective measurement [22]. This is a measurement characterized by an unrevealed outcome, i.e. the system is measured but the outcome is not read out. In a bipartite system \mathcal{SA} composed of subsystems \mathcal{S} and \mathcal{A} , the thermal QD as given by Eq. (10) can be expressed as

$$\mathcal{D}_{th}(S|A) = \min_{\{\Pi_A^k\}} \{I(\rho_{SA}) - I(\Phi_A(\rho_{SA}))\} + [H(\{p_a\}) - S(\rho_A)], \quad (13)$$

where $\Phi_A(\rho_{SA})$ denotes a non-selective measurement $\{\Pi_A^j\}$ on part \mathcal{A} of ρ_{SA} , which reads $\Phi_A(\rho_{SA}) = \sum_j (I_S \otimes \Pi_A^j) \rho_{SA} (I_S \otimes \Pi_A^j)$. In order to derive Eq. (13), we have used that $S(\rho_S|\{\Pi_A^k\}) - S(\rho_S|\rho_A) = I(\rho_{SA}) - I(\Phi_A(\rho_{SA}))$ [12]. Note that Eq. (13) is asymmetric with respect to measurement on \mathcal{S} and \mathcal{A} , which reflects an asymmetry in the roles of system \mathcal{S} and apparatus \mathcal{A} . Due to the asymmetry of QD, a strictly classical bipartite state requires both $\mathcal{D}_{th}(S|A) = 0$ and $\mathcal{D}_{th}(A|S) = 0$. Indeed, this corresponds to a density operator $\rho_{AB} = \sum_{i,j} p_{ij} |i\rangle\langle i| \otimes |j\rangle\langle j|$, where p_{ij} is a joint probability distribution and the sets $\{|i\rangle\}$ and $\{|j\rangle\}$ constitute orthonormal bases for the systems A and B , respectively. For an arbitrary bipartite state composed by subsystems A_1 and A_2 , such strictly classical states can also be identified by a single measure, which is the symmetrized version of QD

$$\mathcal{D}_{th}(A_1 : A_2) = \min_{\Phi} \left\{ I(\rho_{A_1 A_2}) - I(\Phi_{A_1 A_2}(\rho_{A_1 A_2})) + \sum_{i=1}^2 [H(\{p_{a_i}\}) - S(\rho_{A_i})] \right\}, \quad (14)$$

where the measurement operator $\Phi_{A_1 A_2}$ is given by

$$\Phi_{A_1 A_2}(\rho_{A_1 A_2}) = \sum_{j,k} \left(\Pi_{A_1}^j \otimes \Pi_{A_2}^k \right) \rho_{A_1 A_2} \left(\Pi_{A_1}^j \otimes \Pi_{A_2}^k \right). \quad (15)$$

By explicitly using Eq. (6) and the fact that $H(\{p_{a_i}\}) = S(\Phi_{A_i}(\rho_{A_i}))$ we can rewrite Eq. (14) as

$$\mathcal{D}_{th}(A_1 : A_2) = \min_{\Phi} [S(\Phi_{A_1 A_2}(\rho_{A_1 A_2})) - S(\rho_{A_1 A_2})] \quad (16)$$

Eq. (16) provides the thermal generalization of the symmetric QD considered in Ref. [23] and experimentally witnessed in Refs. [24, 25]. The vanishing of $\mathcal{D}_{th}(A_1 : A_2)$ occurs if and only if the state is fully classical. In particular, the absence of $\mathcal{D}_{th}(A_1 : A_2)$ can be taken as the key

ingredient for local sharing of pre-established correlations (local broadcasting) [26].

Generalizations of quantum discord to multipartite states have been considered in different scenarios [12, 27–31], which intend to account for quantum correlations that may exist beyond pairwise subsystems in a composite system. In this direction, one possible approach to account multipartite quantum correlations is to start from the symmetrized QD and then to systematically extend it to the multipartite scenario. This originates GQD as a measure of global quantum discord, as proposed in Ref. [12]. GQD is symmetric with respect to subsystem exchange and shown to be non-negative for arbitrary states [12]. Moreover, it can be detected through a convenient (with no extremization procedure) witness operator [14]. In terms of operational interpretation, GQD may play a role in quantum communication, in the sense that its absence means that the quantum state simply describes a classical probability multidistribution $\sum_{i_1, \dots, i_n} p_{i_1 \dots i_n} |i_1\rangle\langle i_1| \otimes \dots \otimes |i_n\rangle\langle i_n|$ (with $p_{i_1 \dots i_n} \geq 0$, $\sum p_{i_1 \dots i_n} = 1$) and, therefore, allows for local broadcasting [26]. Here we will propose a slightly distinct version of GQD, which will be motivated by an operational interpretation in terms of work extraction in quantum thermodynamics. We will refer to this multipartite measure of quantum correlation as thermal GQD, whose definition is given below.

Definition 1 *The thermal GQD $\mathcal{D}_{th}(A_1 : \dots : A_n)$ for an arbitrary multipartite state ρ composed of subsystems A_1, \dots, A_n is defined as*

$$\mathcal{D}_{th}(A_1 : \dots : A_n) = \min_{\Phi} [S(\Phi_{A_1 \dots A_n}(\rho)) - S(\rho)], \quad (17)$$

where

$$\Phi_{A_1 \dots A_n}(\rho) = \sum_k \Pi_k \rho \Pi_k, \quad (18)$$

with $\Pi_k = \Pi_{A_1}^{j_1} \otimes \dots \otimes \Pi_{A_n}^{j_n}$ denoting a set of local measurements and k an index string $(j_1 \dots j_n)$.

We will now show that the thermal GQD provides an upper bound for the sum of a sequence of bipartite asymmetric thermal discords, which will imply in the interpretation of the thermal GQD in terms of a limit of work extraction through a protocol of local operations in a multipartite system. This is provided by the Theorem below.

Theorem 2 *The thermal GQD $\mathcal{D}_{th}(A_1 : \dots : A_n)$ for an arbitrary multipartite state ρ composed of subsystems A_1, \dots, A_n satisfies the inequality*

$$\mathcal{D}_{th}(A_1 : \dots : A_n) \geq \sum_{i=1}^n \min_{\Phi_{A_1 \dots A_{i-1}}} \mathcal{D}_{th}(\Phi_{A_1 \dots A_{i-1}}(\rho) | A_i), \quad (19)$$

where the asymmetric bipartite contributions $\mathcal{D}_{th}(\Phi_{A_1 \dots A_{i-1}}(\rho)|A_i)$ ($\forall i$) are provided by Eq (13), with $\min_{\Phi_{A_1 \dots A_0}} \mathcal{D}_{th}(\Phi_{A_1 \dots A_0}(\rho)|A_1) \equiv \mathcal{D}_{th}(\rho|A_1)$.

Proof. In Eq. (17), let us consider the difference of joint entropies for a fixed measurement $\Phi_{A_1 \dots A_n}(\rho)$, which yields

$$\mathcal{D}_{\Phi}(A_1 : \dots : A_n) = S(\Phi_{A_1 \dots A_n}(\rho)) - S(\rho). \quad (20)$$

By rewriting $\mathcal{D}_{\Phi}(A_1 : \dots : A_n)$ in terms of the multipartite mutual information, we obtain

$$\mathcal{D}_{\Phi}(A_1 : \dots : A_n) = \left\{ I(\rho) - I(\Phi_{A_1 \dots A_n}(\rho)) + \sum_{i=1}^n [H(\{p_{a_i}\}) - S(\rho_{A_i})] \right\}, \quad (21)$$

where $I(\rho)$ and $I(\Phi_{A_1 \dots A_n}(\rho))$ are generalizations of the mutual information to the multipartite setting [32], which are given by

$$I(\rho_{A_1 \dots A_n}) = \sum_{k=1}^n S(\rho_{A_k}) - S(\rho_{A_1 \dots A_n}), \quad (22)$$

$$I(\Phi_{A_1 \dots A_n}(\rho)) = \sum_{k=1}^n S(\Phi(\rho_{A_k})) - S(\Phi_{A_1 \dots A_n}(\rho)), \quad (23)$$

where

$$\Phi(\rho_{A_k}) = \sum_{k'} \Pi_{A_k}^{k'} \rho_{A_k} \Pi_{A_k}^{k'}, \quad (24)$$

with ρ_{A_k} denoting the marginal density operator for subsystem A_k . In Eq. (21), we now rearrange the terms by adding and subtracting the contributions $I(\Phi_{A_1 \dots A_i}(\rho))$ for all $i \in \{1, \dots, n-1\}$. We then obtain

$$\mathcal{D}_{\Phi}(A_1 : \dots : A_n) = \sum_{i=1}^n \mathcal{D}_{\Phi}(\Phi_{A_1 \dots A_{i-1}}(\rho)|A_i), \quad (25)$$

where

$$\mathcal{D}_{\Phi}(S|A) = I(\rho_{SA}) - I(\Phi_A(\rho_{SA})) + H(\{p_a\}) - S(\rho_A).$$

We can relate Eq. (25) to the thermal GQD through

$$\mathcal{D}_{th}(A_1 : \dots : A_n) = \min_{\Phi} \mathcal{D}_{\Phi}(A_1 : \dots : A_n). \quad (26)$$

This yields

$$\mathcal{D}_{th}(A_1 : \dots : A_n) \geq \sum_{i=1}^n \min_{\Phi} \mathcal{D}_{\Phi}(\Phi_{A_1 \dots A_{i-1}}(\rho)|A_i),$$

which implies in Eq. (19). ■

Remarkably, Theorem 2 provides a relationship between a symmetric measure of quantum correlation (GQD) and a composition of asymmetric operations ($\mathcal{D}_{th}(\Phi_{A_1 \dots A_{i-1}}(\rho)|A_i)$), which involve sequential local measurements over distinct subsystems. In particular,

the local measurements yield a sequence of bipartite records that are chained following a rather simple rule. As an illustration, for a bipartite system ($n = 2$) composed of subsystems A and B , we can write

$$\mathcal{D}_{th}(A : B) \geq \mathcal{D}_{th}(\rho|A) + \min_{\Phi_A} \mathcal{D}_{th}(\Phi_A(\rho)|B), \quad (27)$$

while for a tripartite system ($n = 3$) composed by subsystems A , B , and C , the bound assumes the form

$$\mathcal{D}_{th}(A : B : C) \geq \mathcal{D}_{th}(\rho|A) + \min_{\Phi_A} \mathcal{D}_{th}(\Phi_A(\rho)|B) + \min_{\Phi_{AB}} \mathcal{D}_{th}(\Phi_{AB}(\rho)|C). \quad (28)$$

Moreover, Eq. (19) ensures as a by-product that $\mathcal{D}_{th}(A_1 : \dots : A_n) \geq 0$. Note also that the derivation of Theorem 2 also allows for other less restricted bounds. For instance, suppose we take $\Phi_{A_1 \dots A_n}(\rho)$ in Eq. (25) with local measurement operators Π_k defined by the eigenprojectors of the reduced states ρ_{A_k} , with $k = 1, \dots, n$. This corresponds to the measurement-induced disturbance (MID) basis [33]. For this specific basis, Eq. (25) reads

$$\mathcal{D}_{MID}(A_1 : \dots : A_n) = \sum_{i=1}^n \mathcal{D}_{MID}(\Phi_{A_1 \dots A_{i-1}}(\rho)|A_i).$$

By using that $\mathcal{D}_{MID}(S|A) \geq \mathcal{D}_{th}(S|A)$ for an arbitrary state ρ [21], we can establish

$$\mathcal{D}_{MID}(A_1 : \dots : A_n) \geq \sum_{i=1}^n \min_{\Phi_{A_1 \dots A_{i-1}}} \mathcal{D}_{th}(\Phi_{A_1 \dots A_{i-1}}(\rho)|A_i). \quad (29)$$

Eq. (29) provides an upper bound that is easier to compute than Eq. (19), since no extremization is required to determine $\mathcal{D}_{MID}(A_1 : \dots : A_n)$. However, as we will see, it can be less tight than $\mathcal{D}_{th}(A_1 : \dots : A_n)$.

IV. THERMODYNAMIC INTERPRETATION OF THE THERMAL GQD

The decomposition of the thermal GQD as provided by Eq. (19) allows for a thermodynamic interpretation of the multipartite correlations in terms of work extraction by Maxwell's demons. In particular, Eq. (19) implies that the thermal GQD is an upper bound for a series of bipartite thermal QDs, which are individually related to differences of performances between quantum and classical demons. These bipartite QDs involve locally measured states, which can be associated with a sequence of work extractions in a multi-copy version of the multipartite system. In this direction, we will consider, as a physical resource, n copies of an n -partite quantum system (A_1, A_2, \dots, A_n) . The extraction of work from the system by either a classical or quantum Maxwell's demon will generalize the bipartite protocol through the following procedure:

- The demon *sequentially* takes a subsystem A_i of copy i as a measurement apparatus ($i = 1, \dots, n$).
- The quantum demon can then apply conditional global quantum operations by using the apparatus as a control system. For the classical demon, arbitrary local measurements over the apparatus are allowed.
- For all the subsystems A_j , with $j < i$ (in the copy i), the classical demon also realizes a non-selective measurement such as in Eq. (18) (with no memory cost) in such a way to minimize the difference of its local extracted work at subsystem A_i with respect to the global work extracted by the quantum demon. For the subsystem A_i , a selective local measurement is performed (with memory cost), followed by the effective work extraction from the copy i of the system. A schematic view of this procedure is provided in Fig. 1.

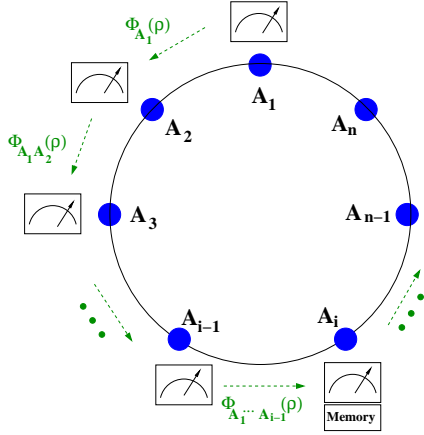


FIG. 1. (Color online) Protocol for a sequential work extraction in a multipartite scenario, where the subsystem A_i is taken as an apparatus at the intermediate step i . For the subsystems A_j , with $j < i$, the classical demon applies a non-selective measurement to optimize the local extracted work at subsystem A_i .

Here we will be interested in the total amount of work that is possible to be extracted by adding the partial work contributions. By extracting work under this procedure, we can then establish an upper bound for the advantage of the quantum demon with respect to the classical demon in terms of the thermal GQD. We begin by taking the system in a general n -partite state ρ and A_1 as the apparatus. Then, by using Eq. (9), we obtain that the difference of work between a quantum demon and the most efficient classical demon is $\Delta W_1 = kT \mathcal{D}_{th}(\rho|A_1)$. For the next step, both classical and quantum demons take the second copy of the system and use A_2 as the apparatus. The classical demon is also allowed to perform a non-selective measurement over A_1 in such a way to minimize the difference with respect to the quantum demon concerning the work extraction from A_2 . Then, we will have $\Delta W_2 = kT \min_{\Phi_{A_1}} \mathcal{D}_{th}(\Phi_{A_1}(\rho)|A_2)$. After n steps,

we denote the total work difference as $\Delta W_t \equiv \sum_i \Delta W_i$. This yields

$$\frac{\Delta W_t}{kT} = \sum_{i=1}^n \min_{\Phi_{A_1 \dots A_{i-1}}} \mathcal{D}_{th}(\Phi_{A_1 \dots A_{i-1}}(\rho)|A_i). \quad (30)$$

Hence, by inserting Eq. (30) into Eq. (19), we obtain that GQD provides an upper bound for the difference ΔW_t of total work between a quantum and a classical Maxwell's demon, i.e.

$$\Delta W_t \leq kT \mathcal{D}_{th}(A_1 : \dots : A_n). \quad (31)$$

As discussed above, an upper (but less strict) bound can be also established in terms of the MID basis, i.e. $\Delta W_t \leq kT \mathcal{D}_{MID}(A_1 : \dots : A_n)$. In both cases, note that the invariance of the upper bound under exchange of subsystems keeps it robust to a change in the order of measurements for the subsystems A_i . Moreover, as we will show, we can illustrate the applicability of Eq. (31) in situations where the bound is rather tight or even is saturated.

V. ILLUSTRATIONS

A. Pure states with Schmidt decomposition

Let us illustrate the upper bound given by Eq. (31) for the quantum advantage of the Maxwell's demon in the case of multipartite pure states $|\psi\rangle$ that admit Schmidt decomposition, whose explicit conditions of existence are discussed in Ref. [34]. We will assume that the system is composed by a set of qubits. In such a case, we can write $|\psi\rangle = \sum_{i=1}^2 \sqrt{p_i} |i_{A_1}\rangle \otimes \dots \otimes |i_{A_n}\rangle$, where $\{|i_{A_k}\rangle\}$ are orthonormal bases, $p_i \geq 0$, and $\sum_i p_i = 1$. For the density operator $\rho_{A_1 \dots A_n} = |\psi\rangle\langle\psi|$, we obtain

$$\rho_{A_1 \dots A_n} = \sum_{i,j=1}^2 \sqrt{p_i p_j} |i_{A_1} \dots i_{A_n}\rangle \langle j_{A_1} \dots j_{A_n}|. \quad (32)$$

Since Schmidt decomposition implies equal spectrum for all single-qubit reduced density operators ρ_{A_k} , we obtain that $S(\rho_{A_k}) = -\sum_{i=1}^2 p_i \log_2 p_i \equiv S$, for any individual subsystem A_k . Therefore, the mutual information is $I(\rho_{A_1 \dots A_n}) = nS$. In order to consider measurements $\Phi(\rho_{A_1 \dots A_n})$ over $\rho_{A_1 \dots A_n}$, it can be shown that, by adopting projective (von Neumann) measurements, the minimization of the loss of correlation is obtained in Schmidt basis, namely, $\{\Pi_{A_k}^i\} = \{|i_{A_k}\rangle\langle i_{A_k}|\}$. This is a consequence of both the group homomorphism of $U(2)$ to $SO(3)$ and the monotonicity of entropy under majorization (see discussion for the state $(|0\dots 0\rangle + |1\dots 1\rangle)/\sqrt{2}$ in Ref. [31]). Then, $\Phi(\rho_{A_k}) = \rho_{A_k}$, which implies $S(\Phi(\rho_{A_k})) = S$. Moreover $\Phi(\rho_{A_1 \dots A_n}) = \sum_{i=1}^2 p_i |i_{A_1} \dots i_{A_n}\rangle \langle i_{A_1} \dots i_{A_n}|$. Therefore, the mutual information after measurement is $I(\Phi(\rho_{A_1 \dots A_n})) = (n -$

1) S and the Shannon entropy $H(\{p_{A_k}\})$ for the subsystem A_k for a measurement in Schmidt basis obeys $H(\{p_{A_k}\}) = S(\rho_{A_k})$. This yields $\mathcal{D}_{th}(A_1 : \dots : A_n) = S$.

As an example, let us consider an n -qubit pure state $|\psi_{A_1 \dots A_n}^{(i)}\rangle = \alpha|0 \dots 0\rangle + \beta|1 \dots 1\rangle$, with $|\alpha|^2 + |\beta|^2 = 1$. In this case, $\mathcal{D}_{th}(A_1 : \dots : A_n) = S = -|\alpha|^2 \log |\alpha|^2 - |\beta|^2 \log |\beta|^2$. The optimal extraction of work by quantum and classical demons are performed through the following strategy: By using a qubit state $|0\rangle_D$ as a memory and A_1 as an apparatus, the quantum demon is able to purify all the individual states of the subsystems A_1, \dots, A_n , while resetting its memory to the ready-to-measure state $|0\rangle_D$. This is obtained through the quantum circuit exhibited in Fig. 2. More specifically, this circuit drives the initial state $|\psi_{A_1 \dots A_n}^{(i)}\rangle \otimes |0\rangle_D$ to the final state

$$|\psi_{A_1 \dots A_n}^{(f)}\rangle \otimes |0\rangle_D = (\alpha|0\rangle + \beta|1\rangle)_{A_1} \otimes |0\rangle_{A_2} \dots |0\rangle_{A_n} \otimes |0\rangle_D. \quad (33)$$

Therefore, since any individual subsystem is in a pure state, we obtain from Eq. (1) that the work W^Q extracted by the quantum demon is

$$W^Q = n k T \log 2. \quad (34)$$

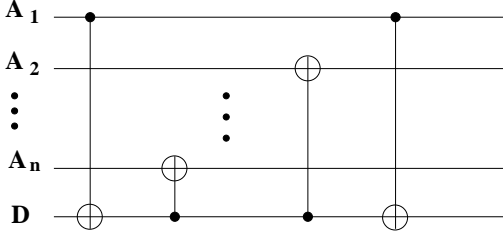


FIG. 2. Quantum circuit for work extraction in the case of an n -qubit pure state $|\psi_{A_1 \dots A_n}^{(i)}\rangle = \alpha|0 \dots 0\rangle + \beta|1 \dots 1\rangle$. The initial state is $|\psi_{A_1 \dots A_n}^{(i)}\rangle \otimes |0\rangle_D$. The quantum demon is able to purify the individual states of the subsystems A_1, \dots, A_n , while resetting its memory to the ready-to-measure state $|0\rangle_D$.

Concerning the classical demon, we assume that one bit of memory is available. Then, a projective local measurement in the computational basis over A_1 can be implemented to purify the individual states of A_1, \dots, A_n all at once. Indeed, through a *selective* measurement over A_1 , the demon obtains

$$|\psi_{A_1 \dots A_n}^{(i)}\rangle \longrightarrow \begin{cases} |0 \dots 0\rangle & \text{(with probability } |\alpha|^2) \\ |1 \dots 1\rangle & \text{(with probability } |\beta|^2) \end{cases} \quad (35)$$

The outcome of this measurement (either 0 or 1) is recorded in the classical memory of the demon, whose state (for an outsider) is a probability distribution that can be described by the classical density operator $\rho_D = |\alpha|^2|0\rangle\langle 0| + |\beta|^2|1\rangle\langle 1|$. At this first step, the classical demon is then able to extract the work $n k T \log 2$, with the energy cost of erasure of the demon's bit given by $k T S(\rho_D) = k T S$. Therefore, we obtain that the

net classical work is given by

$$W^C = n k T \log 2 - k T S. \quad (36)$$

Note that the work W^C in Eq. (36) could also be obtained by decohering the quantum demon's qubit in the quantum circuit of Fig. 2 similarly as discussed for the bipartite case in Ref. [3]. For the next steps ($i > 1$), the classical demon is able to drive the whole system to a fully classical state by performing a suitable non-selective measurement over subsystems A_j , with $j < i$. For example, at step $i = 2$, by non-selectively measuring A_1 in the computational basis, the classical demon obtains the classical probability distribution $\Phi_{A_1}(\rho_{A_1 \dots A_n}^{(i)}) = \alpha^2|0 \dots 0\rangle\langle 0 \dots 0| + |\beta|^2|1 \dots 1\rangle\langle 1 \dots 1|$, with $\rho_{A_1 \dots A_n}^{(i)} = |\psi_{A_1 \dots A_n}^{(i)}\rangle\langle \psi_{A_1 \dots A_n}^{(i)}|$. However, because $\Phi_{A_1}(\rho_{A_1 \dots A_n}^{(i)})$ is already fully classical, the quantum demon cannot obtain any extra advantage from this step on by taking any other A_i ($i > 1$) as an apparatus. Hence, from Eqs. (34) and (36), the total quantum advantage reads

$$\Delta W_t = k T \mathcal{D}_{th}(A_1 : \dots : A_n) = k T S, \quad (37)$$

which saturates the bound in Eq. (31) provided by the thermal GQD.

B. Tripartite Werner-GHZ mixed state

Let us show now that saturation is also possible for mixed composite states. In this direction, we take the a tripartite system ABC described by a Werner-GHZ state, which is given by

$$\rho = \frac{(1-\lambda)}{8} I^{\otimes 3} + \lambda |GHZ\rangle\langle GHZ|, \quad (38)$$

where $I = |0\rangle\langle 0| + |1\rangle\langle 1|$ is the 2×2 identity and $|GHZ\rangle = (|000\rangle - |111\rangle)/\sqrt{2}$, with $0 \leq \lambda \leq 1$. For this state, the minimization of $I(\rho) - I(\Phi_{ABC}(\rho))$ occurs for measurements in the σ_z basis [31]. In this basis, we obtain $H(\{p_a\}) - S(\rho_A) = 0$ (and analogous expressions for B and C). Therefore, from Eq. (21), we can derive that the thermal GQD $\mathcal{D}_{th}(A : B : C)$ is equal to the ordinary GQD (see Refs. [12, 31]), yielding

$$\begin{aligned} \mathcal{D}_{th}(A : B : C) &= \left(\frac{1+7\lambda}{8} \right) \log \left(\frac{1+7\lambda}{8} \right) \\ &+ \left(\frac{1-\lambda}{8} \right) \log \left(\frac{1-\lambda}{8} \right) - 2 \left(\frac{1+3\lambda}{8} \right) \log \left(\frac{1+3\lambda}{8} \right). \end{aligned}$$

As in the previous example, the upper bound for work extraction given by Eq. (31) also saturates, reading

$$\Delta W_t = k T \mathcal{D}_{th}(A : B : C) = k T \mathcal{D}_{th}(\rho|A). \quad (39)$$

For the Werner-GHZ state, the contributions $\mathcal{D}_{th}(\Phi_A(\rho)|B)$ and $\mathcal{D}_{th}(\Phi_{AB}(\rho)|C)$ in Eq. (28) vanish by minimizing Φ with measurements in the σ_z basis, since $\Phi_A(\rho)$ and $\Phi_{AB}(\rho)$ are fully classical states.

C. Tripartite W-GHZ mixed state

We can also consider an example of a mixed state for which saturation is not achieved. To illustrate this, let us consider a tripartite system ABC described by the W-GHZ state

$$\rho = \lambda |W\rangle \langle W| + (1 - \lambda) |GHZ\rangle \langle GHZ|, \quad (40)$$

where $|W\rangle = (|001\rangle + |010\rangle + |100\rangle)/\sqrt{3}$ and $|GHZ\rangle = (|000\rangle - |111\rangle)/\sqrt{2}$, with $0 \leq \lambda \leq 1$. Note that, for

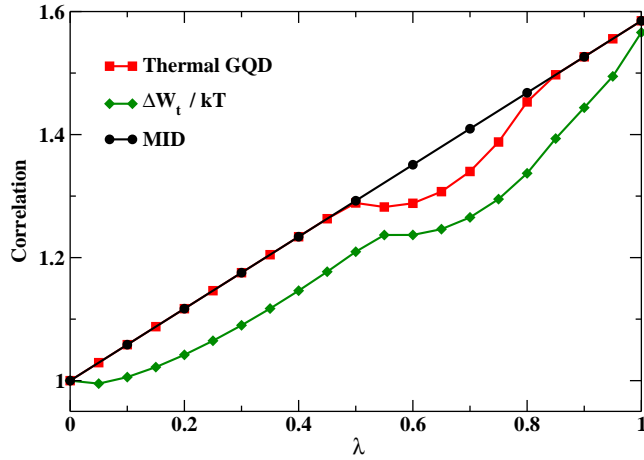


FIG. 3. (Color online) Multipartite MID (black circles), thermal GQD (red squares), and the total quantum advantage ΔW_t for work extraction (green diamonds), as a function of λ for the W-GHZ state. Note that MID is a global upper bound for the other quantities, while the thermal GQD is an upper bound for ΔW_t .

$\lambda = 0$ and $\lambda = 1$, we have pure states, given by $|GHZ\rangle$ and $|W\rangle$ states, respectively. Therefore, by adopting projective measurements, we will have that $\mathcal{D}_{th}(A : B : C) = 1$ for $\lambda = 0$. As λ increases, we numerically find out a monotonic increase of the thermal GQD until $\mathcal{D}_{th}(A : B : C) = \log 3$ for $\lambda = 1$. This can be seen as a consequence of the absence of Schmidt decomposition for the W state, which leaves GQD unconstrained by the entropy of an individual subsystem. For the complete

range of λ , we plot the thermal GQD in Fig. 3 as well as the total quantum advantage $\Delta W_t = \min_i(W^Q - W^{C_i})$. As it can be seen, the bound provided by the thermal GQD does not saturate for the W-GHZ state, but it is considerably tight for all the values of λ . Moreover, we also plot the less restrict bound provided by the multipartite MID $\mathcal{D}_{MID}(A : B : C)$, which is given by a smooth function of λ due to its independence of basis optimization. Note that $\mathcal{D}_{MID}(A : B : C)$ provides a global upper bound for both $\mathcal{D}_{th}(A : B : C)$ and ΔW_t .

VI. CONCLUSIONS

In conclusion, we have analyzed the extraction of thermodynamic work by a Maxwell's demon in a multipartite quantum correlated system. In this direction, we have introduced the thermal GQD as a measure of quantum correlation in a multipartite scenario. Moreover, we have shown that this measure can be applied as an upper bound for the advantage of the quantum demon over its classical counterpart in a protocol of work extraction based on sequential local measurements over n copies of the multipartite state. This result provides therefore a thermodynamic interpretation of the thermal GQD, which can be explored in the context of quantum thermal machines (see, e.g., Ref. [35]). In particular, heat engines driven by the thermal GQD may be investigated as a resource in quantum thermodynamics. In this scenario, it would also be interesting to investigate the quantum advantage of a Maxwell's demon when only a single copy of a multipartite system is available. Moreover, a further relevant topic is the establishment of the conditions for which the proposed thermodynamic protocol may constitute the optimal strategy. We leave these points for future research.

ACKNOWLEDGMENTS

This work is supported by CNPq, CAPES, FAPERJ, and the Brazilian National Institute for Science and Technology of Quantum Information (INCT-IQ).

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